Online learning for audio clustering and segmentation

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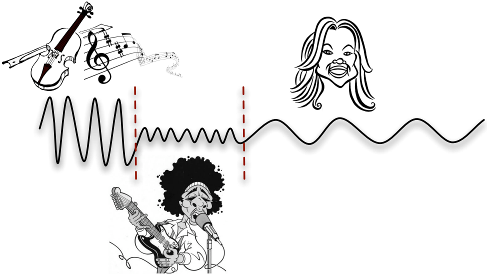
Outline

Audio segmentation

- **Goal**: segment audio signal into homogeneous chunks/segments
- Go from a signal representation to a symbolic representation
- Applications: music indexing, summarization, fingerprinting



Audio segmentation: approaches

- Most existing approaches: find change-points, compute similarities separately
- Change-point detection
  - Use audio features for detecting changes
  - Statistical model on the signal, likelihood ratio tests
- Issues: specific to the task, doesn't use previous parts of the signal, often supervised (needs labeled data)

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- **Hidden (semi-)Markov Models**

Online learning

- Learn a model incrementally, one observation at a time
- Very successful in machine learning, especially large-scale problems
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- **Our goal**: online algorithms for hidden (semi-)Markov models, applications to online audio segmentation and clustering

Outline

Audio signal representation

- Discrete audio signal $x[t] \in \mathbb{R}$
- Short-time Fourier Transform

$$\hat{x}(t, e^{i\omega}) = \sum_{u=-\infty}^{+\infty} x[u] g[u-t] e^{-i\omega u}$$

- Window g (e.g., Hamming), compact support: FFT $\hat{x}_{t,1}, \dots, \hat{x}_{t,p} \in \mathbb{C}$
- $x_t \in \mathbb{R}^p = (|\hat{x}_{t,1}|, \dots, |\hat{x}_{t,p}|)^\top$
- Normalized $\sum_j x_{t,j} = 1$ for invariance to volume

Outline

1. Introduction

2. Representation, models, offline algorithms
   - Audio signal representation
   - Clustering with Bregman divergences
   - Hidden Markov Models (HMMs)
   - Hidden Semi-Markov Models (HSMMs)
   - Offline audio segmentation results

3. Online algorithms
   - Online EM
   - Non-probabilistic algorithm
   - Incremental EM
   - Online audio segmentation results

Bregman divergences

- Euclidian distance doesn't perform well for audio
- Defines a different similarity measure
- Bregman divergence D_ψ for ψ strictly convex:

$$D_\psi(x, y) = \psi(x) - \psi(y) - \langle x - y, \nabla\psi(y)\rangle.$$

- Examples:
  - Squared Euclidian distance $\|x - y\|^2 = D_\psi$ with $\psi(x) = \|x\|^2$
  - KL divergence $D_{KL}(x\|y) = \sum_i x_i \log \frac{x_i}{y_i} = D_\psi(x, y)$ with $\psi(x) = \sum_i x_i \log x_i$

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- Right-type centroid = average (see e.g., (Nielsen and Nock, 2009))

$$\arg\min_c \sum_{i=1}^{n} D_\psi(x_i, c) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Hard clustering (K-means)

- x_i, $i = 1, \ldots, n$, centroids $\mu_1, \ldots, \mu_K$, assignments z_i
- **K-means**, replace $\|x_i - \mu_{z_i}\|^2$ with $D_\psi(x_i, \mu_{z_i})$
  - E-step
$$z_i \leftarrow \arg\min_k D_\psi(x_i, \mu_k) \quad i = 1, \ldots, n$$
  - M-step
$$\mu_k \leftarrow \frac{1}{|\{i : z_i = k\}|} \sum_{i: z_i = k} x_i \quad k = 1, \ldots, K$$

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$$\mu_k \leftarrow \frac{1}{|\{i : z_i = k\}|} \sum_{i : z_i = k} x_i \quad k = 1, \dots, K$$

- Decreases the (non-convex) objective

$$\ell(\boldsymbol{\mu}, \mathbf{z}) = \sum_{i=1}^{n} D_\psi(x_i, \mu_{z_i}).$$

Bregman divergences and exponential families

- Exponential family:

$$p_\theta(x) = h(x)\exp(\langle \phi(x), \theta \rangle - a(\theta))$$

- *Regular* exponential family: minimal, Θ open

$$p_{\psi,\theta}(x) = h(x)\exp(\langle x, \theta \rangle - \psi(\theta))$$

- Bijection between regular exponential families and regular Bregman divergences (Banerjee et al., 2005): $\mu = \nabla\psi(\theta) = \mathbb{E}[X]$,

$$p_{\psi,\theta}(x) = h(x)\exp(-D_{\psi^*}(x, \mu))$$

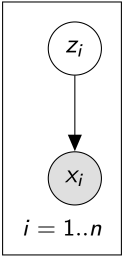
- Example: KL divergence $\Leftrightarrow$ Multinomial distribution

$$h(x)\exp(-\sum_i x_i \log \frac{x_i}{\mu_i}) = h'(x)\prod_i \mu_i^{x_i}$$

Mixture models

- x_i, $i = 1, \dots, n$, K mixture components, emission parameters μ_k
- Model:

$$z_i \sim \pi, \quad i = 1, \dots, n$$
$$x_i | z_i \sim p_{\mu_{z_i}}, \quad i = 1, \dots, n,$$



EM algorithm

- $\mathbf{x}$ observed variables, $\mathbf{z}$ hidden variables, θ parameter
- Goal: maximum likelihood $\max_\theta p(\mathbf{x}; \theta)$

$$\begin{aligned}\ell(\theta) = \log \sum_{\mathbf{z}} p(\mathbf{x}, \mathbf{z}; \theta) &= \log \sum_{\mathbf{z}} q(\mathbf{z}) \frac{p(\mathbf{x}, \mathbf{z}; \theta)}{q(\mathbf{z})} \\ &\geq \sum_{\mathbf{z}} q(\mathbf{z}) \log \frac{p(\mathbf{x}, \mathbf{z}; \theta)}{q(\mathbf{z})}.\end{aligned}$$

- E-step: maximize w.r.t. q. $q(z) = p(z|x; \theta)$
- M-step: maximize w.r.t. θ. $\hat{\theta} = \arg\max_\theta \mathbb{E}_{z \sim q}[\log p(z, x; \theta)]$

Mixture models: EM (soft clustering)

- $x_i, i = 1, \dots, n$, initial parameters π, μ_k.

$$\mathbb{E}_{\mathbf{z} \sim q}[\log p(\mathbf{x}, \mathbf{z}; \pi, \mu)]$$
$$= \sum_i \sum_k \mathbb{E}_q[\mathbb{1}\{z_i = k\}] \log \pi_k + \sum_i \sum_k \mathbb{E}_q[\mathbb{1}\{z_i = k\}] \log p(x_i | k)$$

- E-step

$$\tau_{ik} \leftarrow p(z_i = k | x_i) = \frac{1}{Z} \pi_k e^{-D_\psi(x_i, \mu_k)}$$

- M-step

$$\pi_k \leftarrow \frac{1}{n} \sum_i \tau_{ik}$$
$$\mu_k \leftarrow \frac{\sum_i \tau_{ik} x_i}{\sum_i \tau_{ik}}$$

Outline

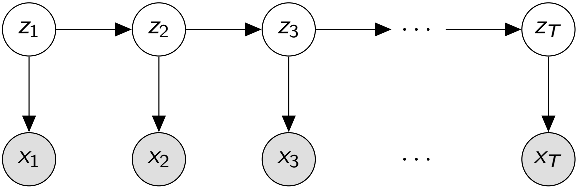
Hidden Markov Models (HMMs)

- Observed sequence $x_{1:T}$, hidden sequence $z_{1:T}$, parameters $\pi, A \in \mathbb{R}^{K \times K}, \mu_k$

$$
\begin{aligned}
z_1 &\sim \pi \\
z_t | z_{t-1} = i &\sim A_i, \quad t = 2, \ldots, T \\
x_t | z_t = i &\sim p_{\mu_i}, \quad t = 1, \ldots, T
\end{aligned}
$$

- Joint likelihood:

$$p(x_{1:T}, z_{1:T}; \pi, A, \mu) = p(z_1; \pi) \prod_{t=2}^{T} p(z_t | z_{t-1}; A) \prod_{t=1}^{T} p(x_t | z_t; \mu)$$



HMM inference: Forward-Backward algorithm

- Inference: compute $p(z_t = i | x_{1:T})$ (*smoothing*)
- Definitions:

$$\alpha_t(i) = p(z_t = i, x_1, \dots, x_t)$$
$$\beta_t(i) = p(x_{t+1}, \dots, x_T | z_t = i).$$

- Recursions, with $\alpha_1(i) = \pi_i p(x_1 | z_1 = i)$, $\beta_T(i) = 1$:

$$\alpha_{t+1}(j) = \sum_i \alpha_t(i) A_{ij} p(x_{t+1} | z_{t+1} = j)$$
$$\beta_t(i) = \sum_j A_{ij} p(x_{t+1} | z_{t+1} = j) \beta_{t+1}(j)$$

- $p(z_t = i | x_{1:T}) \propto \alpha_t(i) \beta_t(i)$

HMM inference: Viterbi algorithm

- Compute *maximum a posteriori* (MAP) sequence:

$$z_{1:T}^{MAP} = \arg\max_{z_{1:T}} p(z_{1:T}|x_{1:T})$$

- Define

$$\gamma_t(i) = \max_{z_1,\dots,z_{t-1}} p(z_1, \dots, z_{t-1}, z_t = i, x_1, \dots, x_t)$$

- Recursion, with $\gamma_1(i) = \pi_i p(x_1|z_1 = i; \mu_i)$:

$$\gamma_{t+1}(j) = \max_i \gamma_t(i) A_{ij} p(x_{t+1}|z_{t+1} = j; \mu_j)$$

- Recover the sequence by storing back-pointers.

HMM learning: EM

- E-step

$$\tau_t(i) \leftarrow p(z_t = i | x_{1:T}) \propto \alpha_t(i)\beta_t(i)$$
$$\tau_t(i,j) \leftarrow p(z_{t-1} = i, z_t = j | x_{1:T}) \propto \alpha_{t-1}(i) A_{ij} p(x_t | j) \beta_t(j)$$

- M-step

$$\pi_i \leftarrow \tau_1(i)$$
$$A_{ij} \leftarrow \frac{\sum_{t\geq 2} \tau_t(i,j)}{\sum_{j'} \sum_{t \geq 2} \tau_t(i,j')}$$
$$\mu_i \leftarrow \frac{\sum_{t\geq 1} \tau_t(i) x_i}{\sum_{t \geq 1} \tau_t(i)}$$

Outline

Duration distributions

- Probability of staying in state i for d time steps:

$$A_{ii}^{d-1}(1 - A_{ii})$$

- i.e., segment lengths follow geometric distributions
- Duration distribution learned implicitely through $A_i i$
- HSMMs: model these duration distributions explicitely (explicit-duration HMM)
- Typical choices: Negative Binomial, Poisson

Hidden Semi-Markov Models

- Segment = (state z, length l), with $l \sim p_z(d)$
- (Markov) transitions A_{ij} between segments
- l i.i.d. observations from cluster z in each segment

$$x_t, \ldots, x_{t+l-1} \sim p_{\mu_z}, \quad i.i.d.$$

Hidden Semi-Markov Models (Murphy, 2002)

- Two hidden variables: state z_t, deterministic counter z_t^D
- $f_t = 1$ iff new segment starts at $t + 1$

$$
\begin{aligned}
p(z_t = j | z_{t-1} = i, f_{t-1} = f) &= \begin{cases} \delta(i,j), & \text{if } f = 0 \\ A_{ij}, & \text{if } f = 1 \text{ (transition)} \end{cases} \\
p(z_t^D = d | z_t = i, f_{t-1} = 1) &= p_i(d) \\
p(z_t^D = d | z_t = i, z_{t-1}^D = d' \geq 2) &= \delta(d, d' - 1),
\end{aligned}
$$

HSMM inference: Forward-Backward algorithm

- Definitions:

$$\alpha_t(j) = p(z_t = j, f_t = 1, x_{1:t})$$
$$\alpha_t^*(j) = p(z_{t+1} = j, f_t = 1, x_{1:t})$$
$$\beta_t(i) = p(x_{t+1:T} | z_t = i, f_t = 1)$$
$$\beta_t^*(i) = p(x_{t+1:T} | z_{t+1} = i, f_t = 1).$$

- Recursions, with $\alpha_0^*(j) = \pi_j$ and $\beta_T(i) = 1$:

$$\alpha_t(j) = \sum_d p(x_{t-d+1:t}|j, d) p(d|j) \alpha_{t-d}^*(j)$$
$$\alpha_t^*(j) = \sum_i \alpha_t(i) A_{ij}$$
$$\beta_t(i) = \sum_j \beta_t^*(j) A_{ij}$$
$$\beta_t^*(i) = \sum_d \beta_{t+d}(i) p(d|i) p(x_{t+1:t+d}|i, d).$$

HSMM: EM

- Define:

$$\gamma_t(i) = p(z_t = i, f_t = 1 | x_{1:T}) \propto \alpha_t(i)\beta_t(i)$$
$$\gamma_t^*(i) = p(z_{t+1} = i, f_t = 1 | x_{1:T}) \propto \alpha_t^*(i)\beta_t^*(i).$$

- E-step

$$p(z_t = i | x_{1:T}) = \sum_{\tau < t} (\gamma_\tau^*(i) - \gamma_\tau(i))$$
$$p(z_t = i, z_{t+1} = j | f_t = 1, x_{1:T}) \propto \alpha_t(i) A_{ij} \beta_t^*(j)$$

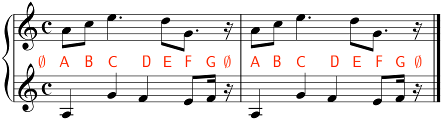
- M-step

$$\pi_i = p(z_1 = i | x_{1:T})$$
$$A_{ij} = \frac{\sum_t p(z_t = i, z_{t+1} = j | f_t = 1, x_{1:T})}{\sum_{j'} \sum_t p(z_t = i, z_{t+1} = j' | f_t = 1, x_{1:T})}$$
$$\mu_i = \frac{\sum_t p(z_t = i | x_{1:T}) x_t}{\sum_t p(z_t = i | x_{1:T})}$$

Outline

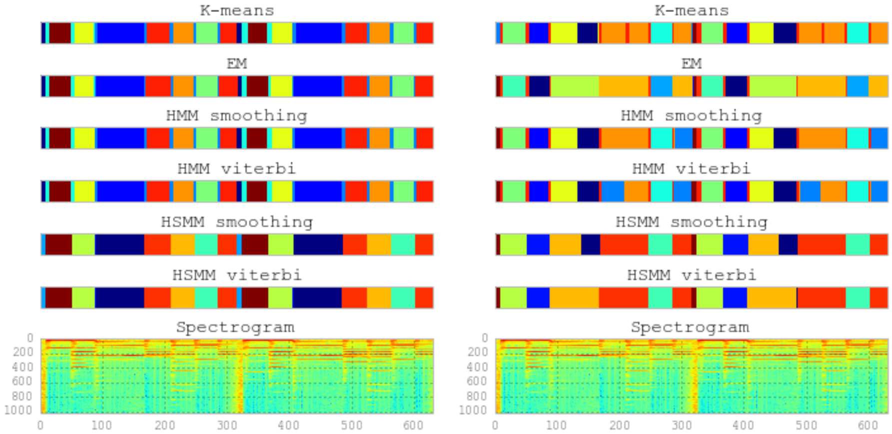
Examples

Ravel, *Ma Mère l'Oye*



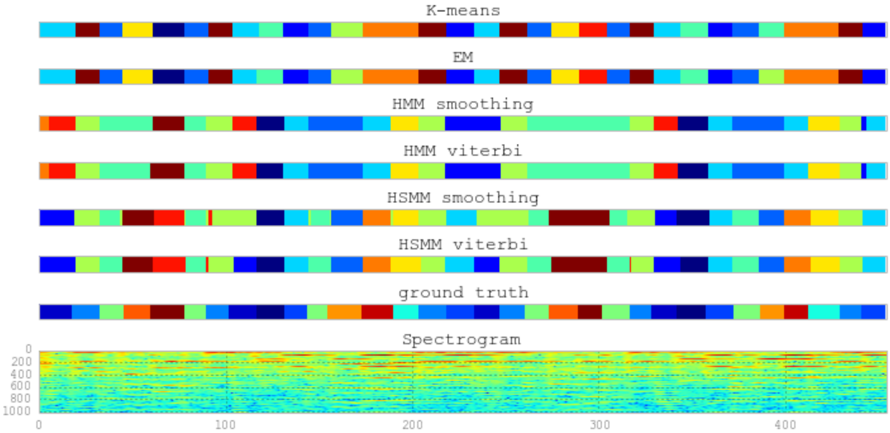
Bach, Violin sonata n. 2, *Allegro*

Results (Ravel)



Different K-means initializations. $K = 9$. HSMM duration distributions fixed to $NegBin(5, 0.95)$.

Results (Bach)



HMM and HSMM randomly initialized (uniform spectrum + noise).
$K = 10$. HSMM durations: $NB(5, 0.2)$ (mean 20).

Outline

Online EM for i.i.d. data (Cappé and Moulines, 2009)

- Complete-data model:

$$p(x, z; \theta) = h(x, z) \exp(\langle s(x, z), \eta(\theta) \rangle - a(\theta))$$

- Batch EM can be written as:

$$S_t = \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}_z[s(x_i, z_i) | x_i; \theta_{t-1}]$$
$$\theta_t = \bar{\theta}(S_t)$$

- Taking the limit $n \to \infty$ (*limiting EM*):

$$S_t = \mathbb{E}_{x \sim P}[\mathbb{E}_z[s(x, z) | x; \theta_{t-1}]]$$
$$\theta_t = \bar{\theta}(S_t).$$

Online EM for i.i.d. data (Cappé and Moulines, 2009)

- Stochastic approximation (Robbins-Monro) procedure to solve $S_{t+1} = \mathbb{E}_{x \sim P}[\mathbb{E}_z[s(x,z)|x; \bar{\theta}(S_t)]]$
- Online EM algorithm:

$$
\begin{aligned}
\hat{s}_t &= (1-\gamma_t)\hat{s}_{t-1} + \gamma_t \, \mathbb{E}_z[s(x_t, z)|x_t; \hat{\theta}_{t-1}] \\
\hat{\theta}_t &= \bar{\theta}(\hat{s}_t).
\end{aligned}
$$

- $\gamma_t = t^{-\alpha}$, $\alpha \in (0.5, 1]$

Online EM for HMMs (Cappé, 2011)

- Complete-data model:

$$p(x_t, z_t | z_{t-1}; \theta) = h(z_t, x_t) \exp(\langle s(z_{t-1}, z_t, x_t), \eta(\theta) \rangle - a(\theta))$$

- Batch EM can be written as:

$$S_k = \frac{1}{T} \mathbb{E}_z \left[\sum_{t=1}^{T} s(z_{t-1}, z_t, x_t) \,\middle|\, x_{0:T}; \theta_{k-1} \right]$$
$$\theta_k = \bar{\theta}(S_k)$$

- *Limiting EM* ($T \to \infty$, with strong assumptions):

$$S_k = \mathbb{E}_{x \sim P}[\mathbb{E}_z[s(z_{-1}, z_0, x_0) | x_{-\infty:\infty}; \theta_{k-1}]]$$
$$\theta_k = \bar{\theta}(S_k),$$

Online EM for HMMs

- Based on the *forward smoothing* recursion
- Define

$$\begin{aligned} S_t &= \frac{1}{t}\,\mathbb{E}_z\left[\sum_{t'=1}^{t} s(z_{t'-1}, z_{t'}, x_{t'}) \,\middle|\, x_{0:t}; \theta\right] \\ \phi_t(i) &= p(z_t = i | x_{0:t}) \\ \rho_t(i) &= \frac{1}{t}\,\mathbb{E}_z\left[\sum_{t'=1}^{t} s(z_{t'-1}, z_{t'}, x_{t'}) \,\middle|\, x_{0:t}, z_t = i; \theta\right] \end{aligned}$$

- We have $S_t = \sum_i \rho_t(i)\phi_t(i)$.

Online EM for HMMs

- Smoothing recursion

$$\phi_{t+1}(j) = \frac{1}{Z}\sum_i \phi_t(i) A_{ij} p(x_{t+1}|z_{t+1}=j)$$

$$\rho_{t+1}(j) = \sum_i \left(\frac{1}{t+1} s(i,j,x_{t+1}) + \left(1 - \frac{1}{t+1}\right)\rho_t(i)\right) r_{t+1}(i|j),$$

with $r_{t+1}(i|j) = p(z_t = i|z_{t+1}=j, x_{0:t})$. Complexity $O(K^4 + K^3 p)$.

Online EM for HMMs

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  with $r_{t+1}(i|j) = p(z_t = i|z_{t+1} = j, x_{0:t})$. Complexity $O(K^4 + K^3 p)$.
- Online EM recursion replaces quantities by estimates, e.g.

$$\hat{\rho}_{t+1}(j) = \sum_i (\gamma_{t+1} s(i, j, x_{t+1}) + (1 - \gamma_{t+1}) \hat{\rho}_t(i)) \, \hat{r}_{t+1}(i|j)$$

- and updates parameters after each observation.

Online EM for HSMMs

- Parameterize HSMM as HMM with 2 hidden variables, z_t and an increasing counter z_t^D

$$p(z_t = j | z_{t-1} = i, z_t^D = d) = \begin{cases} A_{ij}, & \text{if } d = 1 \\ \delta(i,j), & \text{otherwise} \end{cases}$$

$$p(z_t^D = d' | z_{t-1} = i, z_{t-1}^D = d) = \begin{cases} \frac{D_i(d+1)}{D_i(d)}, & \text{if } d' = d+1 \\ 1 - \frac{D_i(d+1)}{D_i(d)}, & \text{if } d' = 1 \\ 0, & \text{otherwise.} \end{cases}$$

- Complexity per observation increased to $O(K^4 D + K^3 Dp)$ instead of $O(K^4 D^2 + K^3 D^2 p)$ thanks to deterministic transitions.

Outline

Objective function from probabilistic models

- Mixture model (with $pi_k = 1/K$)
  - Complete-data likelihood

$$p(\mathbf{x}, \mathbf{z}; \mu) = \prod_{i=1}^{n} p(z_i) p(x_i | z_i; \mu)$$

  - Objective ($= -\log p(\mathbf{x}, \mathbf{z}; \mu) + C$)

$$\ell(\mathbf{z}, \theta) = \sum_{i=1}^{n} D_\psi(x_i, \mu_{z_i})$$

- HMM
  - Complete-data likelihood

$$p(x_{1:T}, z_{1:T}; \mu) = p(z_1) \prod_{t=2}^{T} p(z_t | z_{t-1}) \prod_{t=1}^{T} p(x_t | z_t; \mu)$$

  - Objective

$$\ell(z_{1:T}, \mu) = \frac{1}{T} \sum_{t \geq 1} D_\psi(x_t, \mu_{z_t}) + \frac{\lambda_1}{T} \sum_{t \geq 2} d(z_{t-1}, z_t)$$

Online objective

- Online objective:

$$f_T(\mu) := \min_{z_{1:T}} \ell(z_{1:T}, \mu)$$

- New upper bound (majorizing surrogate) at time t:

$$\hat{f}_t(\mu) := \frac{1}{t} \sum_{i=1}^{t} D_\psi(x_i, \mu_{z_i}) + \frac{\lambda_1}{t} \sum_{i=2}^{t} d(z_{i-1}, z_i)$$

- At time t:
  - $z_{1:t-1}$ fixed from past
  - E-step: $z_t = j = \arg\min_k D_\psi(x_t, \mu_k) + \lambda_1 d(z_{t-1}, k)$
  - M-step: update cluster $\mu_j = \mu_j + \frac{1}{n_j}(x_t - \mu_j)$

Outline

Incremental EM for i.i.d. data (Neal and Hinton, 1998)

- EM = maximize lower bounds

$$f(\theta) = p(\mathbf{x};\theta) \geq \sum_{\mathbf{z}} q(\mathbf{z}) \log \frac{p(\mathbf{x},\mathbf{z};\theta)}{q(\mathbf{z})}.$$

- Maximizer $q(\mathbf{z}) = \prod_i p(z_i|x_i;\theta)$, limit to $\prod_i q_i(z_i)$
- Minorizing surrogates:

$$\hat{f}_n(\theta) = \frac{1}{n}\sum_{i=1}^{n}\sum_{z_i} q_i(z_i)\log\frac{p(x_i,z_i;\theta)}{q_i(z_i)}$$

- Repeat: update single q_i (E-step), maximize $(1/n)\,\mathbb{E}_q[\log p(\mathbf{x},\mathbf{z})]$
- Can be expressed in terms of sufficient statistics

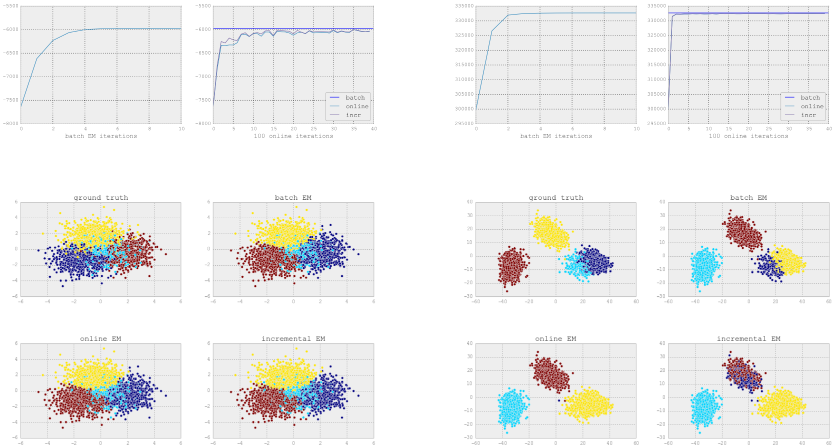
Incremental EM for HMMs

- Only consider lower bounds with $q(z_{1:T}) = q_1(z_1) \prod_{t \geq 2} q_t(z_t | z_{t-1})$
- Surrogates:

$$\hat{f}_T(\theta) = \frac{1}{T} \sum_{t=1}^{T} \left[\sum_{z_{t-1}, z_t} \phi_{t-1}(z_{t-1}) q_t(z_t | z_{t-1}) \log \frac{p(x_t, z_t | z_{t-1}; \theta)}{q_t(z_t | z_{t-1})} \right],$$

  with $\phi_t(z_t) := \sum_{z_{t-1}} \phi_{t-1}(z_{t-1}) q(z_t | z_{t-1})$.
- At time T:
  - $q_{1:T-1}$, $\phi_{1:T}$ fixed from past
  - E-step: $q_T(z_T | z_{T-1}) = p(z_T | z_{T-1}, x_T; \theta)$
  - M-step: $\hat{\theta} = \arg\max_\theta \hat{f}_T(\theta)$

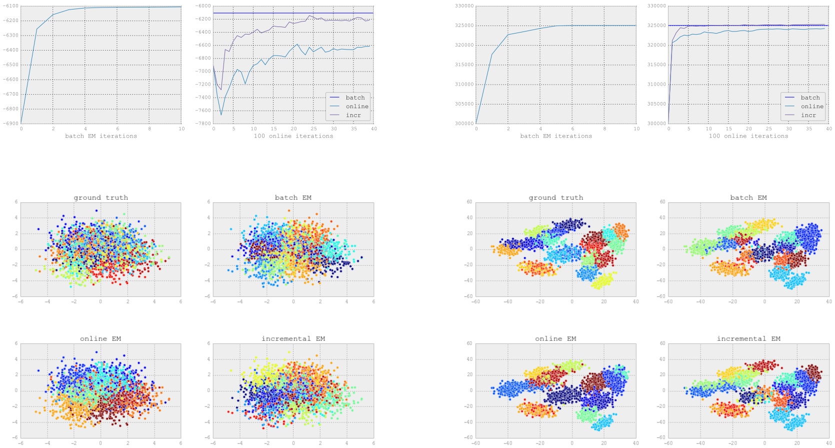
Experiments on synthetic data



Squared Euclidian distance (left) and KL divergence (right).

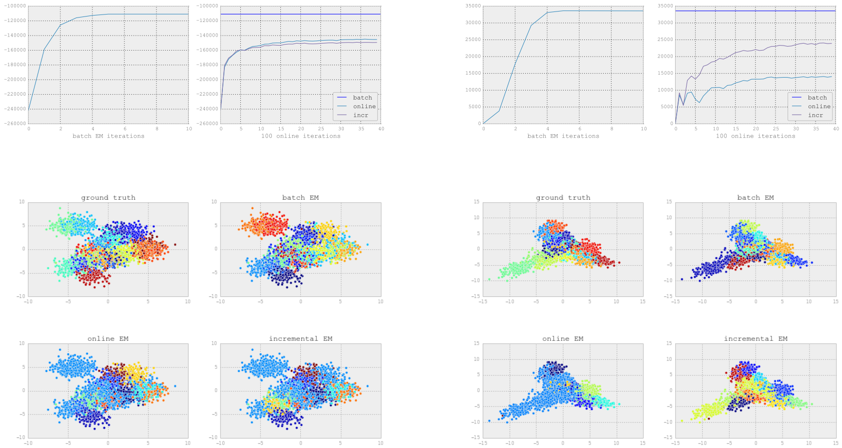
$K = 4,\ p = 5.$

Experiments on synthetic data



Squared Euclidian distance (left) and KL divergence (right).
$K = 20,\ p = 5.$

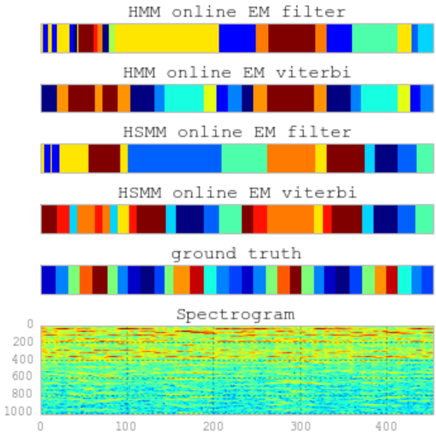
Experiments on synthetic data



Squared Euclidian distance (left) and KL divergence (right).
$K = 20,\ p = 100.$

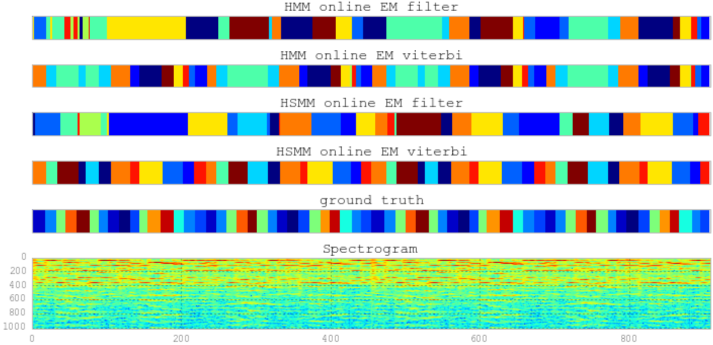
Outline

Online EM for HMM vs HSMM



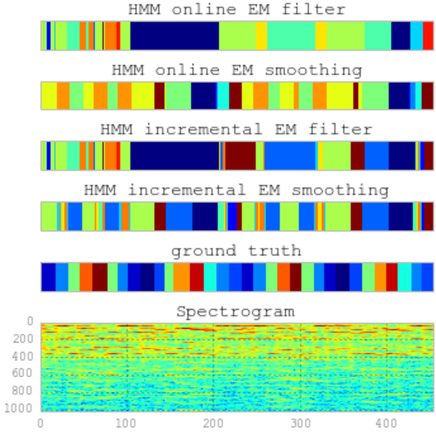
Online EM for HMM/HSMM on Bach. $K = 10$, $NB(30, 0.6)$ (mean 20).

Online EM for HMM vs HSMM

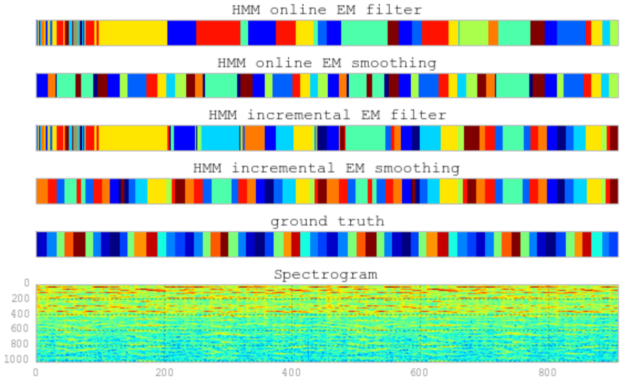


Online EM for HMM/HSMM on Bach. $K = 10$, $NB(30, 0.6)$ (mean 20).

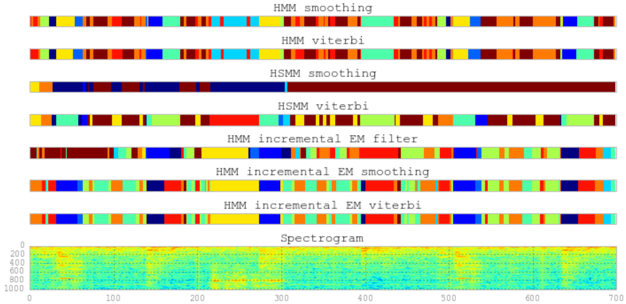
Online vs incremental EM for HMM



Online vs incremental EM for HMM

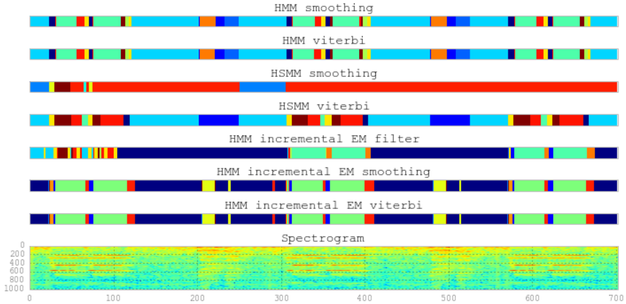


Scenes segmentation



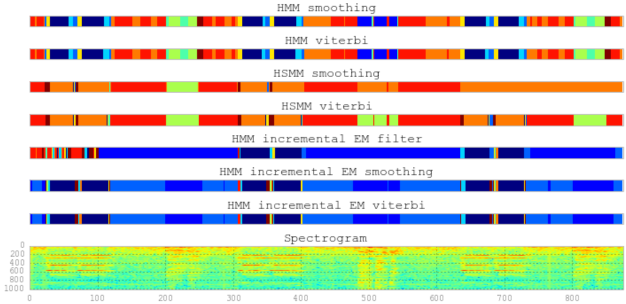
Dropping keys and closing doors (from office live dataset). $K = 10$

Scenes segmentation



Telephone ringing and coughing sounds (from office live dataset). $K = 10$

Scenes segmentation



Telephone ringing and coughing sounds (from office live dataset). $K = 10$

Conclusion

- Joint segmentation and clustering: challenging task
- Offline algorithms perform well
- Harder task for online algorithms, but results improve over time
- Can be used for adaptive estimation (e.g., note templates in *Antescofo* score-following system)
- Main contributions:
  - Extension of online EM algorithm to HSMMs thanks to new parameterization
  - Incremental optimization algorithms for HMMs (EM and non-probabilistic)
  - Applications to audio segmentation, potential improvements in *Antescofo*.

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